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FURTHER MATHEMATICS

9231/14

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



1 Prove by mathematical induction that $\left(\frac{6}{5}\right)^n \geq 1 + \frac{1}{5}n$ for all positive integers n . [5]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

[illegible]

2 The cubic equation $x^3 + bx^2 + cx + d = 0$, where $d \neq 0$, has roots α, β, γ such that $\gamma = \frac{1}{\beta}$.

(a) Show that $\beta + \frac{1}{\beta} = d - b$.

[3]

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(b) Show also that $\beta + \frac{1}{\beta} = \frac{1-c}{d}$.

[2]

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(c) It is given that $b = 3$, $c = -3$ and $d > 0$.

(i) Find the value of d .

[2]

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(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$.

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3 (a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n (r+1)(r+2)(r+3) = \frac{1}{4}n(n^3 + bn^2 + cn + d),$$

where b, c and d are integers to be determined.

[3]

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(b) Express $\frac{2}{(r+1)(r+2)(r+3)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^n \frac{2}{(r+1)(r+2)(r+3)}. \quad [5]$$

(c) State the value of $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+2)(r+3)}$. [1]





4 The points A, B, C have position vectors

$$3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}, \quad 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} + \mathbf{j} - \mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

[illegible]



(b) The point D has position vector $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

Find the shortest distance between the lines AB and CD .

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



5 (a) (i) By writing $\cos\left(\frac{1}{12}\pi\right)$ as $\cos\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right)$, show that $\cos\left(\frac{1}{12}\pi\right) = \frac{1}{4}(\sqrt{6} + \sqrt{2})$. [1]

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(ii) Show also that $\sin\left(\frac{1}{12}\pi\right) = \frac{1}{4}(\sqrt{6} - \sqrt{2})$. [1]

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The matrix $\mathbf{M}$ is such that $\mathbf{M} = \frac{1}{4} \begin{pmatrix} \sqrt{6} + \sqrt{2} & \sqrt{2} - \sqrt{6} \\ \sqrt{6} - \sqrt{2} & \sqrt{6} + \sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(b) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x - y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.





- (c) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

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- (d)** Given that $y = mx$ is an invariant line of the transformation represented by $\mathbf{M}$, show that

$$m^2 \sin\left(\frac{1}{12}\pi\right) + 2m \cos\left(\frac{1}{12}\pi\right) - \sin\left(\frac{1}{12}\pi\right) = 0$$

and find the values of m in the form $a \cot\left(\frac{1}{12}\pi\right) + b \operatorname{cosec}\left(\frac{1}{12}\pi\right)$, where a and b are integers to be determined. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 6** The curve C has polar equation $r = \cos \frac{1}{2}\theta$, for $0 \leq \theta \leq \pi$.

(a) Sketch C .

[2]

In parts **(b)** and **(c)** you may use the identities $\sin \theta \equiv 2 \sin \frac{1}{2} \theta \cos \frac{1}{2} \theta$ and $\cos \theta \equiv 2 \cos^2 \frac{1}{2} \theta - 1$.

- (b)** Find the exact value of the area of the region enclosed by C and the initial line.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- (c) Find the maximum distance of a point on C from the initial line. Give your answer in the form $p\sqrt{q}$, where p and q are rational numbers to be determined. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 7** The curve C has equation $y = \frac{10x^2 - 11x - 18}{10x - 18}$.

(a) Find the equations of the asymptotes of C .

[3]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

(b) Show that C has no stationary points.

[4]

[illegible]

(c) Sketch C , stating the coordinates of the points of intersection with the axes.

[3]

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(d) Sketch the curve with equation $y = \left| \frac{10x^2 - 11x - 18}{10x - 18} \right|$. [2]





- (e) Given that $\left| \frac{10x^2 - 11x - 18}{10x - 18} \right| < 4$ for $\frac{-29 - \sqrt{4441}}{20} < x < p$ and $\frac{-29 + \sqrt{4441}}{20} < x < q$, find the values of p and q . [4]

[illegible]

[illegible]







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